

Elementary Mathematics Notes

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1 Numbers & Their Operations

Types of numbers

Integers (\mathbb{Z}): ..., -3, -2, -1, 0, 1, 2, 3, 4, ...

Prime: integers that are divisible by 1 and itself only, smallest prime number is 2

Rational numbers (\mathbb{Q}) $\frac{\text{integer}}{\text{integer}}$: $\frac{4}{7}$, $-3\frac{1}{8}$, 0.3, $2.\dot{6}\dot{5}$, 92, $\sqrt{16}$

Irrational numbers: π , $\sqrt{2}$, e

Real numbers (\mathbb{R}): all numbers

Standard form

$A \times 10^n$, where n is an integer, and $1 \leq A < 10$

SI prefix

Prefix	10^n
pico	10^{-12}
nano	10^{-9}
micro	10^{-6}
milli	10^{-3}
kilo	10^3
mega	10^6
giga	10^9
tera	10^{12}

Indices

1. $a^m \times a^n = a^{m+n}$

2. $a^m \div a^n = a^{m-n}$

3. $(a^m)^n = a^{mn}$

4. $(ab)^m = a^m b^m$

5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

6. $a^{-n} = \frac{1}{a^n}$

7. $a^0 = 1$

8. $a^{\frac{1}{n}} = \sqrt[n]{a}$

9. $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

2 Ratio & Proportion

Map scale

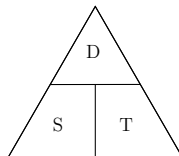
Length scale = 1 : r

Area scale = 1 : r^2

3 Percentage

$$\text{Percentage increase / decrease} = \frac{\text{increase / decrease}}{\text{original}} \times 100\%$$

4 Rate & Speed



$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

5 Algebraic Expressions & Formulae

Special algebraic identities

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

6 Equations

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

7 Set Language & Notation

\in : is an element of

$n(A)$: number of elements in set A

A' : complement of set A

\emptyset : empty set

ξ : universal set

\cup : union

\cap : intersection

\subset : subset

8 Problems In Real-World Contexts

Simple interest

$$I = \frac{PRT}{100}$$

Compound interest

$$A = P \left(1 + \frac{R}{100}\right)^n$$

9 Angles, Triangles & Polygons

Types of polygons

No. of sides	Polygons
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon

Sum of interior & exterior angles

Sum of interior angles = $(n - 2) \times 180^\circ$

Sum of exterior angles = 360°

10 Congruence & Similarity

Congruent & Similar Triangles

Congruent triangles	Similar triangles
SSS, SAS, AAS, RHS	SSS, SAS, AAA

Ratio of area & volume

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

11 Pythagoras' Theorem & Trigonometry

Pythagoras' theorem

$$a^2 + b^2 = c^2$$

Trigonometric ratios

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

TOA CAH SOH is applicable for only right-angled triangles

Obtuse angles

$$\sin(180^\circ - \theta) = \sin \theta$$

$$\cos(180^\circ - \theta) = -\cos \theta$$

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

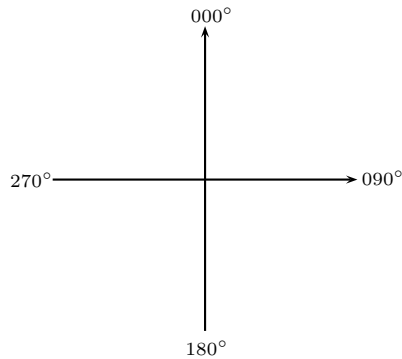
Cosine rule

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Area of triangle

$$\text{Area of triangle} = \frac{1}{2}ab \sin C$$

Bearings



A bearing is a **3-digit positive number** with units of degree to show direction **clockwise** from the north direction.

12 Mensuration

Conversion

$$1 \text{ m} = 100 \text{ cm}$$

$$1 \text{ m}^2 = 10,000 \text{ cm}^2$$

$$1 \text{ m}^3 = 1,000,000 \text{ cm}^3$$

Radian & Degree

$$180^\circ = \pi \text{ rad}$$

Arc length & sector area

Degree

$$s = \frac{\theta}{360^\circ} \times 2\pi r, \text{ where } \theta \text{ is in degrees}$$

$$A = \frac{\theta}{360^\circ} \times \pi r^2, \text{ where } \theta \text{ is in degrees}$$

Radian

$$s = r\theta, \text{ where } \theta \text{ is in radians}$$

$$A = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

13 Coordinate Geometry

Cartesian coordinate

(x, y)

Gradient

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

Equation

$$y - y_1 = m(x - x_1)$$

$$y = mx + c$$

*Vertical line: $x = a$

*Horizontal line: $y = b$

Length

$$\text{Length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

14 Vectors In 2 Dimensions

Representation

Vectors can be represented by $\begin{pmatrix} x \\ y \end{pmatrix}$, \vec{AB} , \mathbf{a} or \underline{a} .

Magnitude

$$|\vec{AB}| \text{ or } |\mathbf{a}| = \sqrt{x^2 + y^2}$$

15 Data Analysis

Mode

Mode is the **most frequently occurring** number. A set of data can have **more than one** mode.

Mean

$$\text{mean} = \frac{\text{sum of all numbers}}{\text{number of numbers}}$$

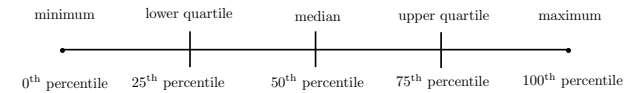
Median

Median is the **centre** number when the numbers are arranged from **smallest to largest**.

Range

$$\text{Range} = \text{maximum} - \text{minimum}$$

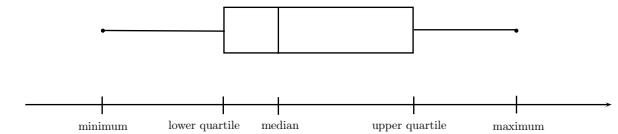
Quartiles & percentiles



Interquartile Range

Interquartile range = upper quartile – lower quartile

Box-and-whisker plot



Mean & standard deviation

Ungrouped

$$\text{Mean, } \bar{x} = \frac{\sum x}{N}$$

$$\text{Standard deviation, } \sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{N}}$$

Grouped

$$\text{Mean, } \bar{x} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation, } \sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$